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(6) STATISTICAL CHARACTERIZATION OF ~~ALTITUDE~~ Altitude MATRICES

BY COMPUTER.

REPORT 3.

The effect of resolution on gradients calculated from an altitude matrix.

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To DR. H. LEMONS

Chief Scientist, European Research Office, U.S. Army

(11) for 1975

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REPORT THREE

The effect of resolution on gradients calculated

for an altitude matrix

AND....

Can we circumvent the stationarity problem ?

by IAN S EVANS

CONTENTS

THE EFFECT OF RESOLUTION ON GRADIENTS CALCULATED FROM AN ALTITUDE MATRIX

AIM A basic problem in geomorphometry is that any measurement varies with scale, i.e. with the extent of the area or line involved. Terrains cannot be compared if they have been analysed at different scales. At present our knowledge of the variation of measurements with scale is inadequate for any decision on the spatial scale or scales at which comparison of terrains would be most meaningful. Hence further information on such variation is urgently required.

The overall aim of this project is to produce compact and efficient techniques for geomorphometry from altitude matrices. There are no technical limitations on the application of such techniques to altitude matrices at any available scale, though at very coarse scales the curvature of the earth introduces map projection problems. Nor is there any great difficulty in generating such matrices. However, results do vary with scale and the possibility exists that in the real world certain scales of analysis are more meaningful than others. The question of scale therefore cannot be ignored in geomorphometry.

The aim of the research reported here is to assess the effect of data resolution in terms of grid mesh - the separation of data points in a square altitude matrix - on gradient, which is the geomorphometric parameter with most direct process implications. As resolution becomes coarser, we expect information on surface details to be lost, leaving only the more generalised features : hence mean gradient as measured from the matrix will be reduced. Three questions should be resolved: (i) how may the decline in mean gradient with scale be summarised? (ii) is there a range of scales over which fairly constant mean gradients are obtained? (iii) are there any peculiar effects at the finest scale, due to data errors or inadequacies?

DATA Data from two sources were used in this section of the investigation; in both, altitude is resolved to the nearest metre. Three 100 x 100 altitude matrices were taken from UNAMACE automatically-profiled data for the Cache area of Oklahoma. These data had been edited and lightly smoothed by the CONPLOT program. Their matrix resolution (grid mesh) is 25m in both directions, and they are referred to as CACHE 1, CACHE 2 and CACHE 3. Each covers an area of 2.5 x 2.5 km.

A fourth matrix covers part of the Torridon mountain area in northwest Scotland and is less satisfactory in that altitudes were read manually from maps. These maps, however, were the excellent new 1/10,000 (and 1/10,560) Ordnance Survey maps with all contours photogrametrically surveyed, at 25-feet intervals. Quite small errors in altitude can have serious effects on gradients and aspects calculated from several altitude data points, and even greater effects on plan and profile curvatures, but the selected resolution of 100m is coarse relative to the detail shown on the maps. The TORRIDON matrix is also 100 x 100, and therefore covers an area 10 x 10 km. Its inclusion provides a quite different type of topography, mountains with glacial cirques and troughs.

TECHNIQUE At this stage in the project, gradient is calculated by the simplest possible technique, that of finite differences in altitude between the four neighbouring matrix points, north and south, and east and west. The altitude of the central point, to which the calculated gradient is allotted, is not used. This technique provides gradients for all the original data points except those in the outermost rows and columns. A 100 x 100 altitude matrix yields a 98 x 98 gradient matrix : a 50 x 50 yields 48 x 48, and so on.

Given a full data matrix, it is easy to simulate the results which would have been obtained if the matrix covering the same area had been generated with coarser resolution. This is done by 'thinning', by taking

every second point in both directions, or every third point, or every fourth, and so on. Obviously the number of remaining data points diminishes rapidly as thinning becomes more extreme. With 100 x 100 matrices, the greatest thinning applied was every twenty-fifth point. This left a 4 x 4 altitude matrix, and only a 2 x 2 gradient matrix.

Calculation of gradient for thinned matrices permits mean gradient (or other gradient statistics) to be plotted against matrix resolution. Points for coarser resolutions, however, are less reliable due to small sample sizes. Their instability is reduced by taking 'all possible' thinned matrices and averaging all gradients calculated therefrom. There are four possible matrices in which every second point of the original has been taken : one has its top left (northwest) corner at the top left corner of the original matrix, the point (1,1); another, at the second point in the top row, (1,2); a third, at the left of the second row, (2,1); and a fourth, at the second point in the second row, (2,2). Each of these thinned matrices has 50 x 50 data points, but for some levels of thinning the size of the thinned matrix varies somewhat with position; e.g. every third point can produce 34 x 34 <u>or</u> 33 x 33.

Although results from 'all possible' thinned matrices are more stable, it must not be forgotten that adjacent altitudes are not independent of each other, and this autocorrelation reduces effective sample size from its apparent value. Calculating mean gradient for each thinned matrix gives some impression of the variability of results with position, for that size of matrix.

<u>MEANS FOR INDIVIDUAL THINNED MATRICES</u> All results obtained to date show the expected decline of mean gradient with increasing (widening) grid mesh. Table 1 provides a representative set of results for individual thinned matrices. It applies to those thinned matrices which are centred, as far as possible, within the original matrix, that is which omit near-equal numbers of rows or columns at each border.

Figure 1 represents these results on arithmetic graph paper. All four data sets provide concave-up plots, but the first 6 to 12 points are near-linear except for Cache 1, where the concavity is concentrated at fine meshes. The mild oscillations at coarse meshes are attributed to sampling variability (the varying incidence of grids with topography).

The decline of mean gradient with increasing grid mesh is most marked for Torridon, and least for Cache 1. The three Cache matrices, however, clearly form a family, with a trend of increasing gradient from Cache 1 through 2 to 3.

The four plots are sufficiently similar to offer the hope that all may tend to straight lines with some transformation. The concave-up nature of these plots suggests use of a logarithmic transformation of one or both axes. Further analysis, however, may be based more soundly by taking gradients from thinned matrices in all possible distinct positions, not just centrally-located ones.

MEANS FOR ALL POSSIBLE THINNED MATRICES The sample size is increased (more so in appearance than in reality) by taking all possible thinned matrices. For a given degree of thinning, all calculated gradients are then averaged. (This is not quite the same as averaging the mean gradients from different thinned matrices, as the number of data points and hence the number of gradients may vary slightly with matrix position relative to the original matrix).

Results corresponding to those in Table 1 are given in Table 2 on this new basis. Figures 2 and 3 show that these trends are more stable than those for individual thinned matrices. One remaining anomaly in all three Cache plots is the high value for 550m mesh (every 22nd point from the original matrix) compared with 500m (every 20th point). The divergent upper and lower lines for each mean gradient plot show the maximum and minimum mean gradient for any individual thinned matrix. (Table 3). These show that, until only every fourteenth point is taken, variation

with position within the original matrix is less than variation with grid mesh.

There is some consistency in the position of those matrices yielding maximum and minimum mean gradients. The greatest differences between maximum and minimum means are found for Cache 2, of which the heterogeneity is noted below. Every 20th point (500m mesh) gives relatively small differences because all the gradient matrices are square (3 x 3): a similar effect is present to some extent for every 2nd, 4th, 5th, 10th and 25th. Conversely, taking every 13th, 17th or 21st point gives thinned matrices of varying size and shape, so that differences between thinned matrices are increased.

Figures 2 and 3 are plotted with a logarithmic scale of gradient and they provide closer approximations to straight-line plots than do the purely arithmetic plots of Figure 1. This is reasonable, since it is expected that mean gradient will never be zero or negative however coarse the grid mesh. The Cache plots are still mildly concave-up, especially that for Cache 1. The Torridon plot is initially straight, mildly convex around 800m, and rather more concave around 1,600m.

Deviations from linearity remain sufficient to prompt investigation of different transformations. In Figure 4, the results of the previous two figures are plotted with a logarithmic scale of grid mesh, but an arithmetic scale of gradient. The results for Cache are good: matrix 1 plots much less concave-up than before, while plots for 2 and 3 are now mildly convex-up. For Torridon, however, the convexity is exaggerated while the concavity remains. Hence, such a plot is unsatisfactory, apart from the point that a linear trend would predict negative mean gradients at extreme grid meshes.

Finally, a logarithmic transformation of both axes is applied (Figure 5). Again, the results for Cache are good; even the plot for Cache 1 is near-linear. It is now, however, slightly convex-up, and the other two

Cache plots are more so. The Torridon plot is rather similar to the previous one, much less straight than the log-gradient plot.

The information to hand is conflicting as to the nature of the relationship between mean gradient and grid mesh; further examples would be useful. Provisionally, the plots of logarithmic gradient against arithmetic grid mesh are used, but it may be that double logarithmic plots are of more general applicability. The former imply an exponential relationship, the latter, a power relationship. At this stage, then, no precise mathematical expressions are fitted to the trends.

Extrapolation of the Torridon trend to a mesh of 1.5m, for comparison with field measurements, suggests a mean gradient of some 15.7 degrees. Extrapolation of the Cache trends suggests 6.1, 3.5 and 2 to 3 degrees for matrices 3, 2 and 1 respectively. Such extrapolation,however, does not account for information lost between points in the original grid. The process of generating a grid creates an implicit surface which is usually smoother than the real-world one, so the gradient estimates are minimum values. Such a loss of information might cause convexity of the plot of gradient against the finest meshes in the series, but such an effect is not very apparent here. The convexity of the double-logarithmic plots affects almost the whole series, and is unlikely to be due to this effect.

Comparison is more safely made in terms of a 100m grid mesh, for which the Torridon mean gradient is 14.8 degrees, and Cache 4.88, 2.77 and 0.98 degrees respectively.

STANDARD DEVIATIONS For Torridon, standard deviations of gradient are plotted against grid mesh in Figure 3; for Cache, in Figure 6. The linearity of these plots compares with that for mean gradient plots. They are, however, steeper, showing a greater senstivity of standard deviation to grid mesh: much of the gradient variability is at a fine scale and is lost by coarsening the mesh.

Standard deviations are roughly in proportion to means except that those for Cache 2 are unusually high, and exceed mean gradients for all but the coarsest resolutions. This may be because the Cache 2 matrix is a mixture of plains and foothills.

CONCLUSIONS

(i) Mean gradient may decline exponentially as grid mesh is increased. However there is some difference between the Torridon and the Cache results and a power function might be equally appropriate.

(ii) The decline in mean gradient is continuous. There is no range of scales over which mean gradient remains fairly constant.

(iii) No particular effect at the finest scales, attributable to data errors or inadequacies, is identified.

ACKNOWLEDGEMENTS

I thank Iain Bain for generating the Torridon matrix, and Robert P. Macchia and Richard Clark of E.T.L. for supplying the Cache data, from which the three matrices were taken.

Table 1 Average gradient (degrees) per single thinned, centred matrix, as a function of grid mesh.

GRID MESH AS MULTIPLE OF ORGINAL MATRIX MESH	CACHE 3	CACHE 2	CACHE 1	TORRIDON
1	5.86	3.30	1.47	15.48
2	5.51	3.11	1.24	14.15
3	5.13	2.94	1.10	12.93
4	4.80	2.78	0.99	11.77
5	4.52	2.62	0.93	10.78
6	4.03	2.47	0.89	9.72
7	3.85	2.36	0.82	8.74
8	3.70	2.21	0.78	7.48
9	3.52	2.10	0.79	7.19
10	3.39	2.03	0.75	5.96
11	3.19	1.89	0.70	6.04
12	3.15	1.75	0.67	5.07
13	3.13	1.63	0.64	4.10
14	2.88	1.85	0.64	3.70
15	2.75	1.68	0.60	3.79
16	2.86	1.49	0.60	3.81
17	2.97	1.19	0.73	3.03
18	2.97	1.46	0.68	3.08
19	2.84	1.48	0.64	2.49
20	2.64	1.21	0.50	2.85
21	2.04	0.65	0.66	2.73
22	1.92	0.72	0.64	3.12
23	2.04	0.85	0.67	2.80
24	2.11	0.96	0.60	2.52
25	2.09	1.28	0.58	1.93

Table 2 Average gradient (degrees) for all possible points in the original matrix, as a function of grid mesh.

GRID MESH AS MULTIPLE OF ORIGINAL MATRIX MESH	100 x 100 matrix name (and original mesh,m) TORRIDON (100)	CACHE 1 25	CACHE 2 25	CACHE 3 25	GRID MESH,m (Cache)	NO. OF GRADIENTS CALCULATED
1	14.76	1.47	3.30	5.86	25	9604
2	13.73	1.23	3.11	5.54	50	9216
3	12.82	1.08	2.93	5.20	75	8836
4	11.98	0.98	2.77	4.88	100	8464
5	11.17	0.91	2.61	4.59	125	8100
6	10.32	0.86	2.47	4.33	150	7744
7	9.56	0.81	2.34	4.12	175	7396
8	8.77	0.77	2.23	3.92	200	7056
9	7.97	0.74	2.14	3.76	225	6724
10	7.19	0.71	2.06	3.60	250	6400
11	6.37	0.68	1.98	3.46	275	6084
12	5.70	0.65	1.90	3.31	300	5776
13	5.01	0.63	1.85	3.26	325	5476
14	4.49	0.61	1.78	3.11	350	5184
15	4.07	0.59	1.71	3.02	375	4900
16	3.71	0.57	1.67	2.94	400	4624
17	3.47	0.56	1.64	2.83	425	4356
18	3.33	0.55	1.58	2.79	450	4096
19	3.15	0.54	1.55	2.70	475	3844
20	3.01	0.51	1.56	2.58	500	3600
21	2.93	0.51	1.60	2.56	525	3364
22	2.87	0.52	1.59	2.53	550	3136
23	2.84	0.51	1.55	2.48	575	2916
24	2.79	0.50	1.51	2.44	600	2704
25	2.68	0.49	1.46	2.38	625	2500

TABLE 3 Range of mean gradient between different thinned matrices, as a function of grid mesh

Grid Mesh As Multiple of Original Matrix Mesh	Torridon 100m	Cache 1 25m	Cache 2 25m	Cache 3 25m	No. of Gradients per Matrix	No. of thinned Matrices
	100 x 100 matrix name and original grid mesh (metres)					
1	-	-	-	-	9604	1
2	13.681-13.790	1.219-1.244	3.068-3.148	5.508-5.570	2304	4
3	12.643-12.948	1.050-1.112	2.798-3.034	5.095-5.319	961-1024	9
4	11.846-12.107	.966-1.004	2.639-2.887	4.791-4.988	529	16
5	10.980-11.321	.895- .940	2.464-2.747	4.462-4.701	324	25
6	10.102-10.614	.831- .895	2.191-2.733	4.115-4.480	196- 225	36
7	9.228-10.098	.760- .854	2.045-2.639	3.891-4.290	144- 169	49
8	8.128-9.283	.739- .816	1.910-2.557	3.661-4.193	100- 121	64
9	7.418-8.972	.697- .780	1.844-2.433	3.496-4.022	81- 100	81
10	6.219-8.097	.644- .771	1.751-2.346	3.253-3.871	64	100
11	5.576-7.031	.606- .745	1.574-2.377	3.085-3.770	49- 64	121
12	4.777-6.247	.602- .708	1.363-2.334	2.993-3.693	36- 49	144
13	4.080-6.303	.555- .694	1.217-2.486	2.805-3.787	25- 36	169
14	3.620-5.518	.546- .678	1.365-2.405	2.582-3.616	25- 36	196
15	3.028-5.035	.503- .664	1.129-2.601	2.596-3.340	16- 25	225
16	2.853-5.569	.475- .626	1.080-2.428	2.559-3.267	16- 25	256
17	2.636-4.705	.432- .635	.764-2.462	2.552-3.438	9- 16	289
18	1.983-4.673	.421- .635	.861-2.441	2.304-3.362	9 - 16	324
19	1.708-4.797	.421- .636	.877-2.445	2.141-3.203	9- 16	361
20	1.781-4.588	.414- .622	.779-2.273	2.113-2.851	9	400
21	0.981-4.846	.387- .700	.459-2.951	2.091-3.344	4- 9	441
22	1.121-5.584	.372- .676	.459-2.781	1.892-3.297	4- 9	484
23	1.144-5.265	.382- .653	.563-2.699	1.862-3.232	4- 9	529
24	1.336-4.726	.344- .634	.524-2.875	1.613-3.145	4- 9	576
25	1.385-4.803	.301- .616	.515-2.949	1.713-3.075	4	625

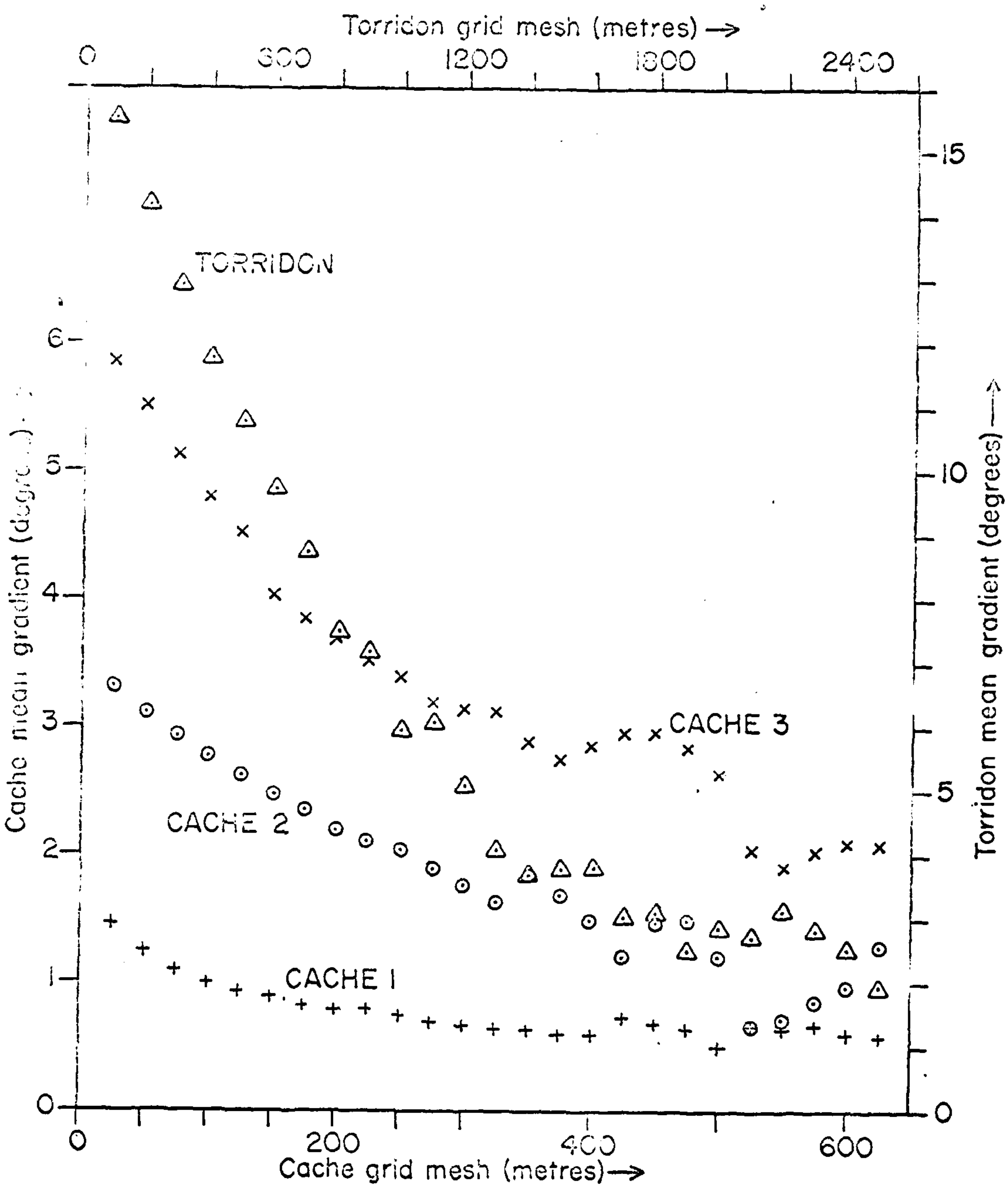

Figure 1 : Mean gradients for single thinned, centred matrices, as functions of grid mesh . Results for Torridon are plotted at half the vertical scale and one-quarter the horizontal scale of those for Cache .

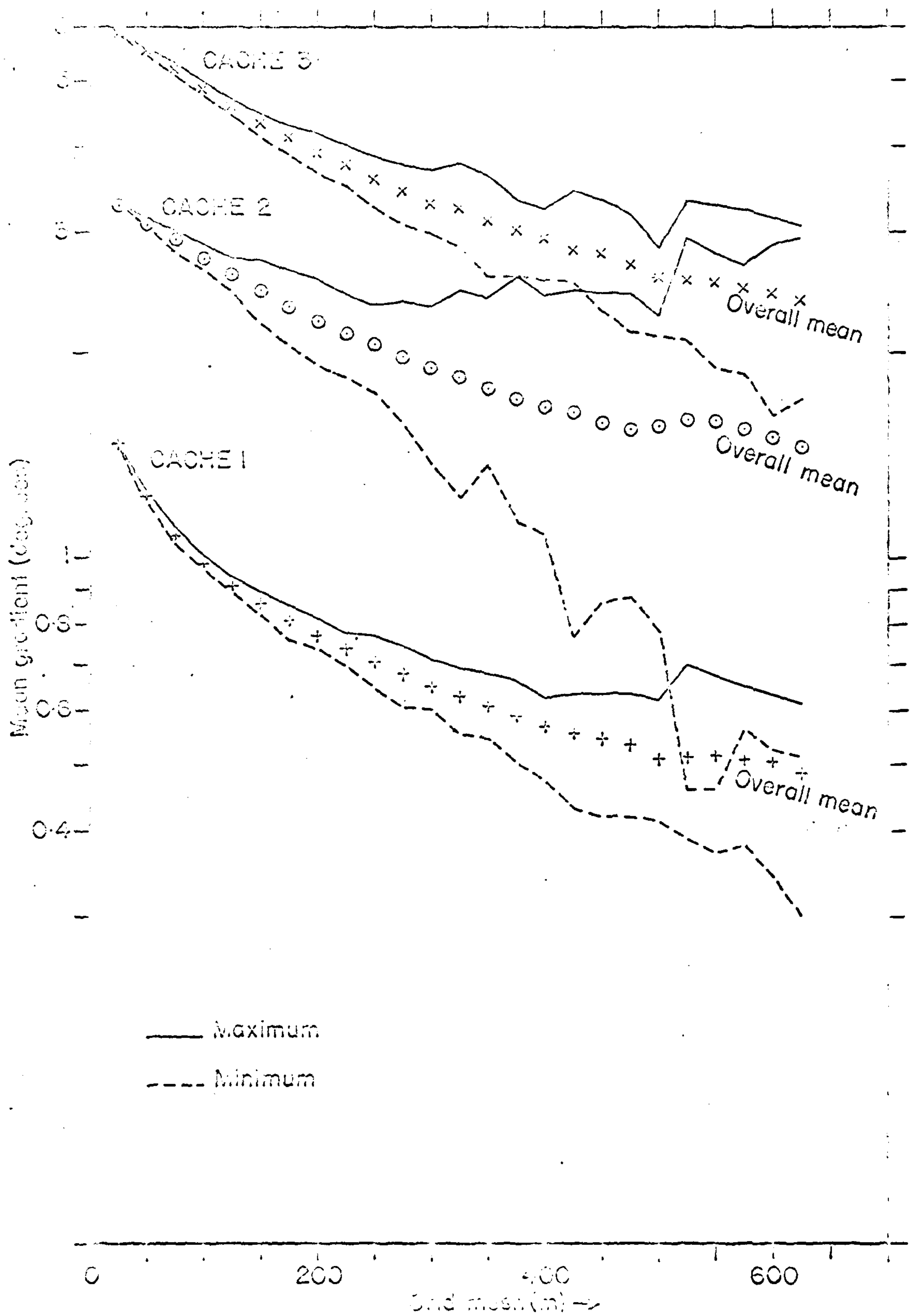

Figure 2 : Cache mean gradients as functions of grid mesh : logarithmic scale of gradient

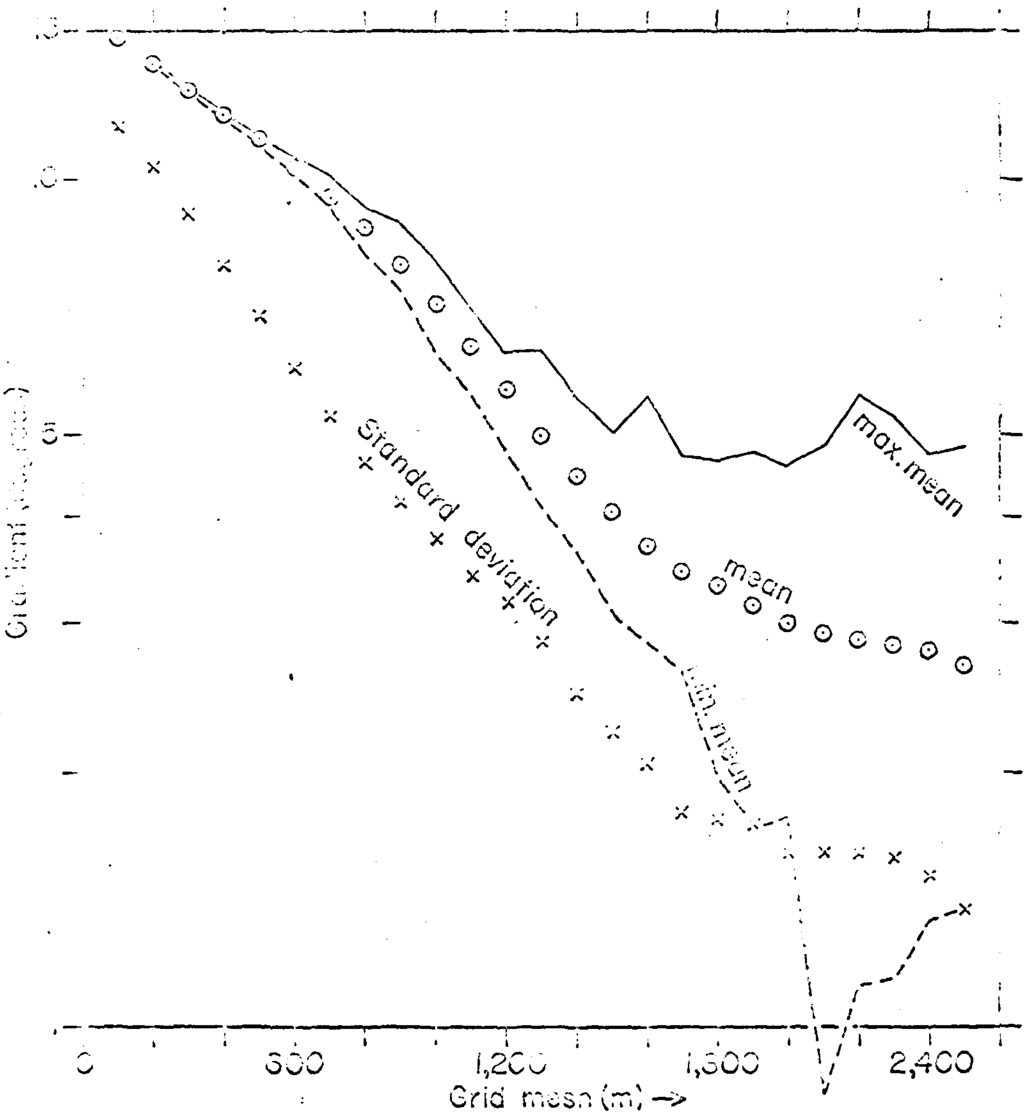

Figure 3 : Torridon gradient statistics as functions of grid mesh; logarithmic scale of gradient

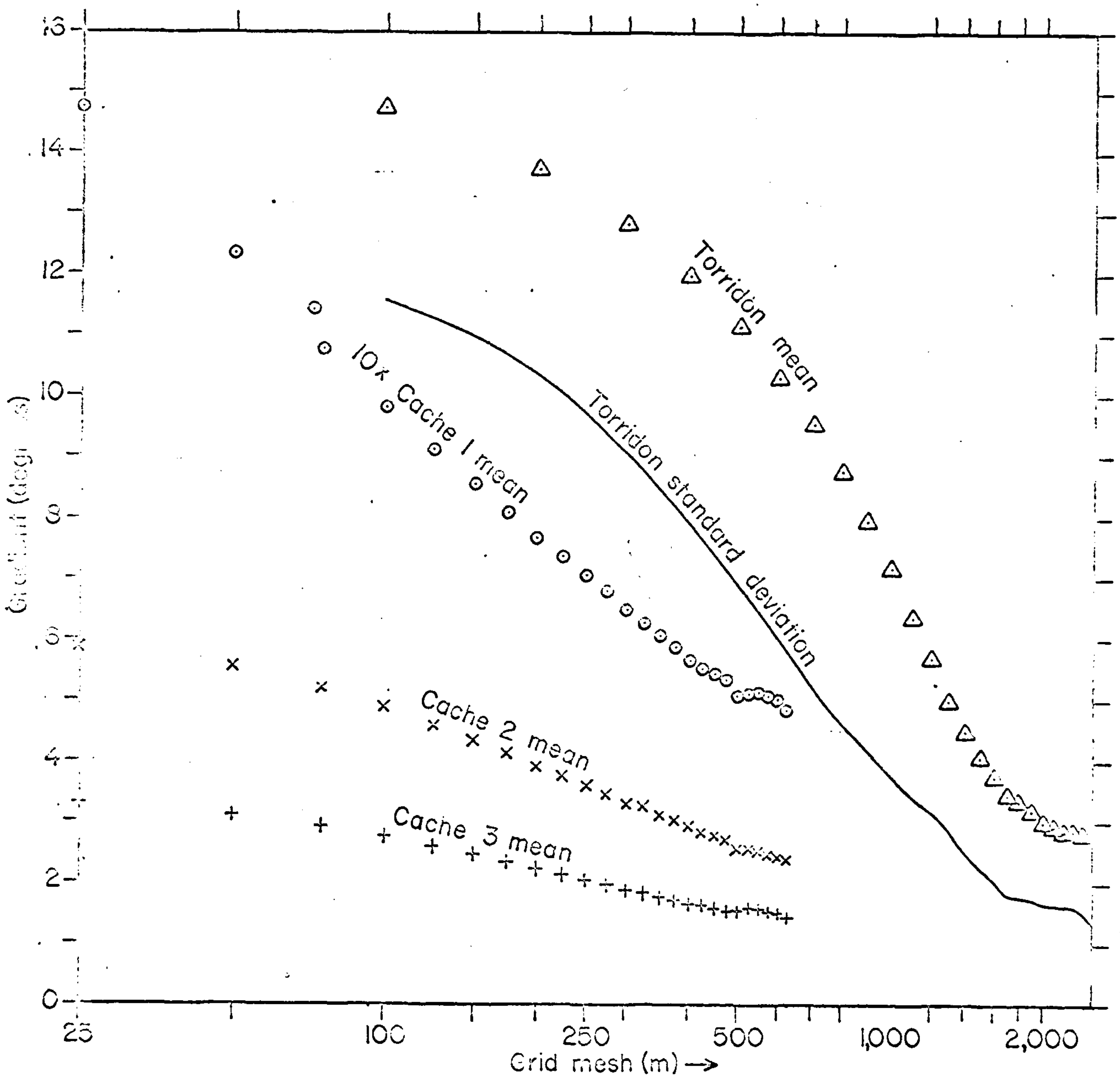

Figure 4 : Gradient statistics as functions of grid mesh; logarithmic scale of grid mesh

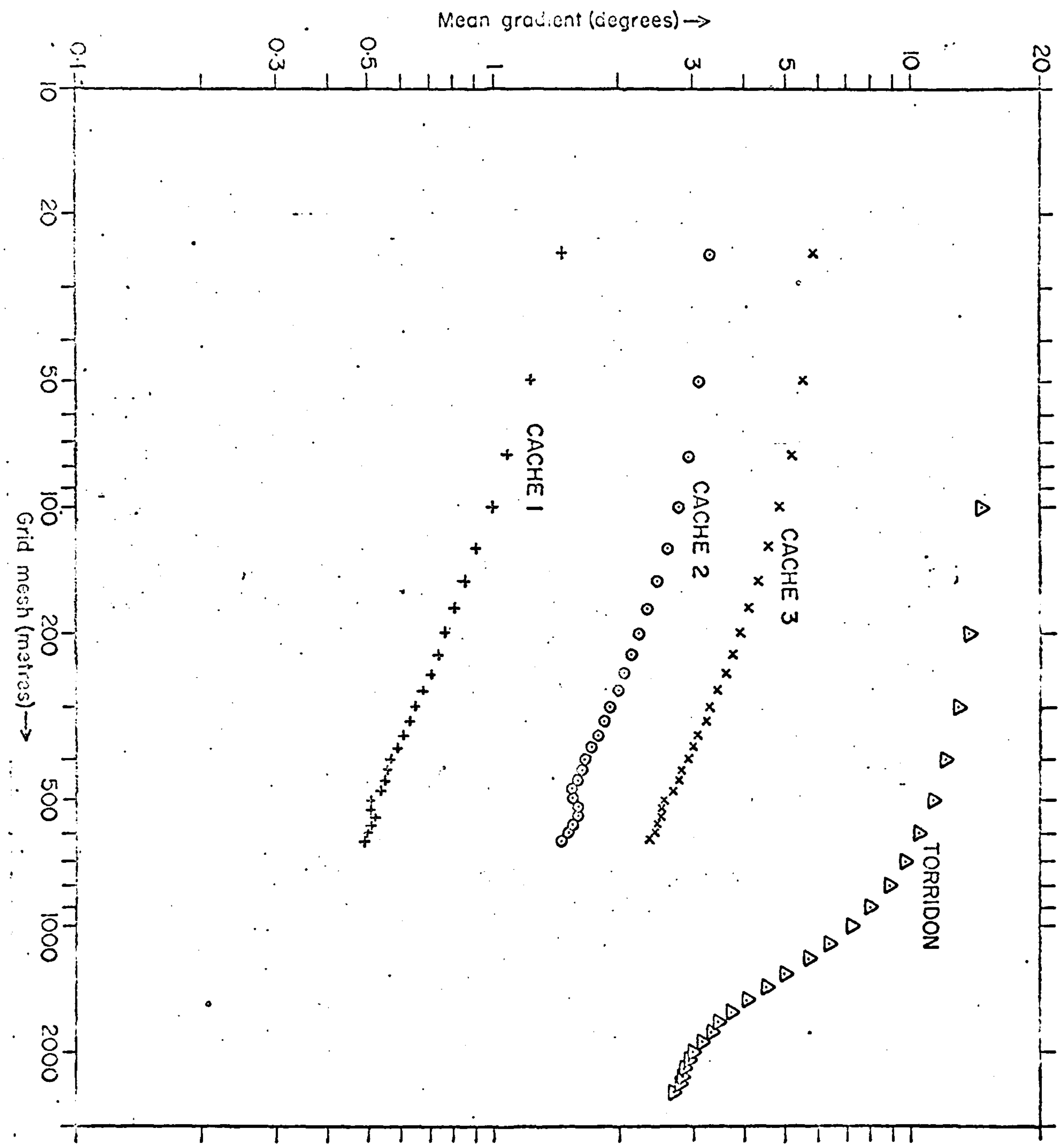

Figure 5 : Mean gradient as a function of grid mesh, both on logarithmic scales .

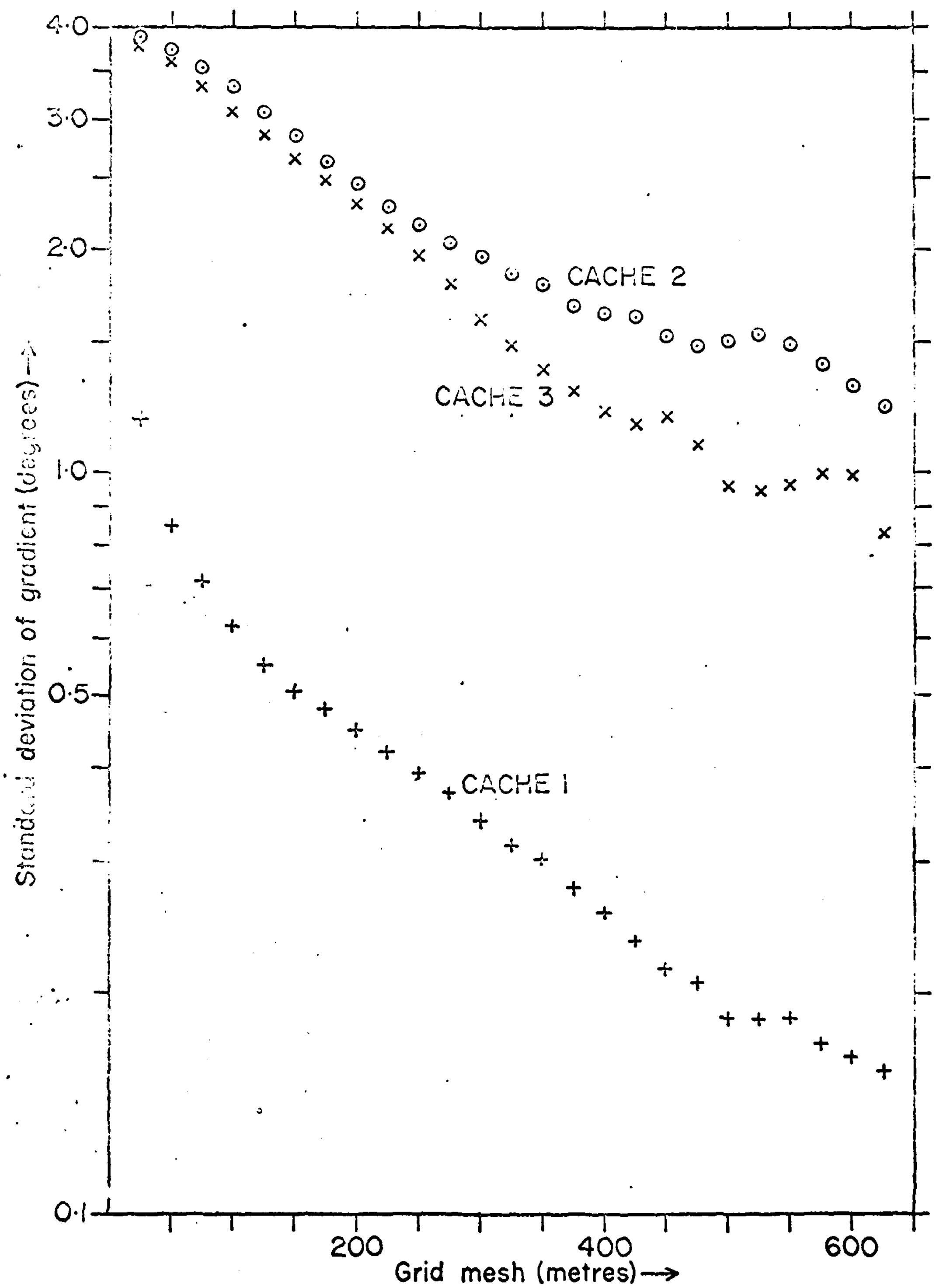

Figure 6 : Standard deviation of gradient for Cache matrices, as a function of grid mesh ; logarithmic scale of gradient .

Appendix (relating to the theme of Report 2)

CAN WE CIRCUMVENT THE STATIONARITY PROBLEM?

Geographers are increasingly realising (Hepple, 1974) that their basic statistical problem is autocorrelation, the non-independence of measurements in relation to their position in a spatial series. Spatial models must incorporate the autoregressive tendences of their subject-matter. Going one step further and applying techniques such as autocorrelation to spatial series, we encounter the problem of stationarity, a statistical requirement which is far more pervasive than we have yet realised. In fact, we should recognise that non-stationarity is the main problem in spatial series analysis, while assessment of non-stationarity is a neglected topic.

A series in time or space is non-stationary if any of its statistical properties vary significantly with position in the series (cf. Blackman and Tukey, 1959, p. 4-6; Jenkins and Watts 1968, p.147-152). The most obvious example is where the average value of a sub-series varies steadily over a series, so that a linear or curvilinear trend is present. This can easily be removed, but more complex types of non-stationarity involving higher moments of sub-series frequency distributions are less tractable. For example, differences in variance may be corrected by variate transformation only if they are closely and positively related to differences in mean. Even more complex are spatial variations in the spectrum from one side of a series to another; either the relative importance of different wavelengths, or characteristic wavelengths themselves, may vary. Given the large data requirements of even a single spectral analysis in two dimensions, investigation of spatial variations in spectra poses great difficulties; yet we expect that such variations will be present in most spatial series .

After recording, quite correctly, this limitation of spectral analysis, Hepple (1974) went on to discuss alternative techniques. Since the stationarity requirement received little further mention, the reader might draw the implication that these weaker and less comprehensive techniques do at least have greater robustness, in that stationarity is not required. Yet each of them is based on stationarity of relevant properties of the series.

Firstly, filtering out certain wavelengths of variability (Bassett and Norcliffe, 1969) would be useless if the wavelengths varied across the series to which a uniform filtering was applied. Filtering assumes stationarity of spectral shape. Secondly, simple tests for the presence or absence of autocorrelation based on contiguity in a regular or irregular lattice (e.g. Cliff and Ord, 1973) are averaged over the whole lattice (spatial series). If the spectrum varies considerably, it is conceivable that positive autocorrelation in one region may be cancelled out by negative autocorrelation in another, leaving the false impression that autocorrelation is absent. There are often regional variations in areal or population size of subdivisions which form an irregular lattice, making variations in the magnitude (if not the sign) of autocorrelation rather likely.

Thirdly, autoregressive models (Tobler, 1969) are based on correlation coefficients averaged map-wide, either within the same series or with the corresponding spatial series for the preceding time. Hence they are meaningful only if systematic variations in autocorrelation (i.e. in the spectrum) are absent.

With irregular spatial subdivisions, residuals from autoregressive models may be non-stationary in variance solely because of variations in subdivision size. But it should not be assumed that the non-stationarity comes from this cause and no other. We are on safer ground with regular spatial series, where stationarity of mean and variance of residuals can be tested directly.

Fourthly, trend surfaces (power series or Fourier) are usually estimated by least squares, involving most of the assumptions of the linear model if the

coefficients are to be best linear unbiased estimators. Of these, constant variance of the error term is the most likely to be violated, and autocorrelation of the errors is also quite likely. Hence although trend surfaces are useful in removing certain simple forms of non-stationarity, they are themselves based on assumptions of stationarity of variance, and its distribution without concentrations at any wavelength.

Fifthly, cascaded averaging and differencing (Curry, 1971; Cox, 1973) or the 'analysis of scale-variance' (Moellering and Tobler, 1972) at first sight offer a robust, empirical approach to spectral decomposition. Yet non-stationarity in the mean will be incorporated into the high-level, broad-scale variance components. All the terms are averaged map-wide, so the assumption is made that the spectrum (of scale components) is constant throughout the spatial series. (Curry, 1970, also suggested mapping within-unit variance at each scale: this would portray non-stationarity, but it would not provide a test. No examples of such maps have been published). It should be noted that this technique is essentially that of calculating a 'pilot spectrum', as described some time ago by Blackman and Tukey (1959, p.45-47 and 135-139). They recommended it as a quick technique, helpful in making further decisions for analysis of the spectrum, but not as an alternative to full spectral analysis.

Finally, Evans (1972) suggested the empirical description of areas of various sizes by parameters for frequency distributions of surface height, and of the first and second vertical and horizontal derivatives of the surface. Even this technique is based on averaging within areas; it assumes homogeneity of each frequency distribution within the area it describes.

Are there, then, any techniques which do not require some sort of stationarity of the series? Even calculations of the mean or of overall variance are less meaningful if the property involved varies over the surface. Clearly stationarity is involved in any statistical description of a series. In other words, we describe series by averaged properties; hence we must choose series within which these properties are reasonably stationary, so that

averages are meaningful.

Let us return to the question of whether spectral analysis is more demanding of stationarity than are other techniques. Its requirement of longer series is clearly a disadvantage, since stationarity is less likely than in short series. Also, it requires stationarity of mean, variance, and frequency components; only in the first case is non-stationarity easily corrected. Hence its requirements are more exhaustive. This is not surprising, since spectral analysis is much more ambitious than the calculation of a trend surface, a first-order autocorrelation (or 'contiguity') coefficient, or a low-order autoregression. It is concerned with estimating many components, not just a few. In this perspective, the stationarity requirements for spectral analysis are relatively no more demanding than for other series techniques. Each technique requires stationarity of the relevant properties, over a series of length adequate for their calculation.

Tests for stationarity of mean and variance are available. Tests for stationarity of frequency components or of autocorrelation over various lags should also be investigated. Since non-stationarity is difficult to demonstrate in small data sets, some effort should be devoted to its investigation in large spatial series. These are available, at least, for altitudes in the U.S.A.

Spectral analysis does of course have other drawbacks. It requires a number of fairly arbitrary, even subjective decisions (Evans and Bain, 1973, 1974). These involve the type of detrending, the extent of smoothing at the edges of the series, the number of zeros to be added around the edges (after the mean has been set to zero), and the number of estimates required (the size of classes in the frequency domain). Even though package programs are available (Rayner and McCalden, 1972) and can produce quite precise, unbiased results (Evans and Bain, 1974), spectral analysis is not a routine technique which can be entrusted to a novice. Good decisions on control parameters for the analysis can be made only after considerable experience, and even experts come to different decisions. There is a certain circularity in attempting to produce as smooth

a spectrum as possible; many peaks are sidelobes (i.e. spurious), but there is no foolproof way of distinguishing these from true peaks in the spectrum. Both Jenkins (1961) and Tukey (1961) recommended repeated computer runs with different control parameters.

Other criticisms of spectral analysis are more tenuous. Inferential techniques are available for continuous series (Blackman and Tukey, 1969, p.21-25; Rayner, 1972, p.25; p.98-100), although they depend heavily on assumptions of normality and stationarity (Jenkins 1961). Suppression of asymmetry (treating east-west in the same way as west-east) is not a serious drawback, since it applies to most spatial series techniques, and asymmetry is easily detected by the techniques of circular statistics.

Tukey (1961) suggested that although non-stationarity prevents a spectrum from being a complete statistical specification of a series, it should not discourage us from making estimates of an average spectrum. This should be supplemented by study of further relevant characteristics, such as variation in mean, variance and spectrum across a series. We should obtain series large enough for spectral analysis to be performed on subsets. These could overlap, and variations in significant parameters could be mapped. They could even provide evidence for single-variable regionalisation into zones of homogeneous patterns of variability.

The same analyses could be regarded as tests for stationarity, and areas with similar spectra could be amalgamated; the larger data base would permit greater resolution of the spectrum. Similar considerations apply to all the techniques discussed above. This approach emphasises the critical importance of delimitation of the area to which any spatial technique is to be applied, which brings us back to a traditional geographic problem.

The number of arbitrary decisions required by spectral analysis may justify use of weaker techniques which require fewer initial decisions, but we should not deceive ourselves that these techniques circumvent the problem of stationarity. This is omnipresent in spatial series and, as Granger (1969) suggested, spatial

series are much less likely to be stationary than are time series. If the limited possibilities of detrending do not provide stationarity of the requisite order, the only solution may be to map our problem out of the spatial domain; for example, to use functions of cost or time distance or of flow statistics as the controlling variables. Even so, residuals must be non-autocorrelated, not just globally as provided for by the present tests, but at all scales and in all parts of a series.

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REFERENCES

BASSETT, K.A. and G.B. NORCLIFFE — 1969, Filter theory and filter methods in geographical research; Dept. of Geography, University of Bristol, Seminar Paper A 20, 31 pp.

BLACKMAN, R.B. and J.W. TUKEY — 1959, The measurement of power spectra from the point of view of communications engineering; Dover Publications (New York), 190pp.

CLIFF, A.D. and J.K. ORD — 1973, Spatial autocorrelation, Pion (London)

COX, N.J. — 1973, The analysis of topographical matrices; B.A. Dissertation 73-15, Dept. of Geography, University of Cambridge.

CURRY, L. — 1970, Univariate spatial forecasting; Economic Geography 46 p. 241-258.

CURRY, L. — 1971, Applicability of space-time moving-average forecasting p.11-24 (and 456-457) in M. Chisholm, A.E. Frey and P. Haggett (Eds)., 'Regional Forecasting' Butterworth (London), Proc. of Colston Symposium.

EVANS, I.S. — 1972, General geomorphometry, derivatives of altitude, and descriptive statistics; pp.17-90 in 'Spatial Analysis in Geomorphology', R.J. Chorley (Ed.), Methuen (London).

EVANS, I.S. and I. BAIN — 1973 and 1974 'Statistical characterization of altitude matrices by computer' Report 1 (1973) and 2 (1974) on grant DA-ERO-591-73-G0040.

GRANGER, C.W.T. — 1969 Spatial data and time series analyses; p.12-4 in 'Studies in Regional Science', A.J. Scott (Ed.), Pion (London).

HEPPLE, L. — 1974, The impact of stochastic process theory upon spatial analysis in human geography. Progress in Geography, No. 6, p. 89-142. Arnold (London)

JENKINS, G.M. — 1961, General considerations in the analysis of spectra; Technometrics 3 (2) p.133-166, reprinted as p.98-131 in

E. Parzen (Ed.) 'Time Series Papers', 1967, Holden-Day.

JENKINS, G.M. and D.G. WATTS — 1968, Spectral Analysis; Holden-Day (San Francisco).

MOELLERING, H. and TOBLER — 1972, Geographical variances; Geographical Analysis, 4 (1), p.34-50.

RAYNER, J.N. — 1971, An introduction to spectral analysis; Pion (London), 194pp.

RAYNER, J.N. and G. McCALDEN — 1972, Computing programs for cross-spectral analysis in two dimensions; Ohio State University, Dept. of Geography, Discussion Paper 27, 66pp.

TOBLER, W. — 1969, Geographical filters and their inverses; Geographical Analysis, 1, p.234-253.

TUKEY, J.W. — 1961, Discussion, emphasising the connection between analysis of variance and spectrum analysis; Technometrics 3 (2), p.191-219.

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